Solutions to Exercises from Absorption Chillers and Heat Pumps

# Chapter 2.

## 2.1. The reversible COP of refrigeration cycle.

### 2.1.a. Partials

Lest the notation seem confusing, recall the following synonym pairs: , , , where . Then from equation 2.11 or identically 2.22,

### 2.1.b. Sensitivity plot

For COP centered at , and derivatives.

|  |
| --- |
|  |
|  |

### 2.1.c. Design solution to improve COP.

No constraints? Intuitively, we would want to move toward the pole where evaporator and condenser are at the same temperature. However, this is not a practical end goal. Also, in realistic cycle, we really want to move toward matching equation 2.22. Based on equation 2.11 alone, with respect to magnitude of small temperature changes, the performance increases fastest by adjusting condenser temperature downward. This is why adding a wet cooling tower pays off fast in a dry climate. However, assuming temperature for heat rejection is fixed, per unit change, increasing evaporator temperature increases COP “faster” than increasing heat input temperature.

## 2.2. The COP of a reversible absorption heat pump.

Not interesting, because of equation 2.12.

## 2.3. The COP of a Type II absorption heat pump (heat transformer).

From equation 2.14,

Here are some plots, centered around ,

|  |
| --- |
|  |
|  |

Interesting in this case is that decreasing the ambient heat rejection temperature monotonically increases efficiency (compared to the opposite relation in refrigeration cycle), because it is decoupled from the heat pump side of the cycle. If that is fixed, then with respect to magnitude of small temperature changes, increasing heat input temperature increases performance faster than decreasing heat output temperature. But there is nothing said here that is not obvious from exergy …

## 2.4. Entropic average temperature of a sensible heat transfer process with constant specific heat.

We use equations 2.16, except that there should be a discussion of signage; let us take Q and S to be positive for heat transferred out of the system. Also, note that “assume the process is reversible” means to neglect entropy generation from finite rate heat transfer, relevant only the first (left) equation, and all other sources of entropy generation. In other words, the system boundary includes only the water, which is taken to be perfectly mixed. Therefore, for sensible heating,

For instance, with , we have .

## 2.5. Entropic average temperature of heat transfer including phase change.

This is the same process illustrated in figure 2.6. Again we are just integrating. In general this can be done numerically using fluid lookup tables. Alternatively, we can approximate the specific heat of liquid and vapor close to saturation. In that case, we have

For de-superheating and subcooling, we use the same formula as in exercise 2.4. For a pure condensing process at constant pressure and temperature, , and , where is the enthalpy of vaporization for the given pressure and is the change in vapor quality, in this case .

Getting some values for water, we can calculate, for this process.

## 2.6. Zero-order model for heat exchange: size of evaporator

The approach here will be to implement example 2.2 in open software, as a test case. Before beginning optimization, it is good to discuss the approach to solving the system of equations for temperatures and heat flows, given a set of heat transfer coefficients. Particularly, given the form of the lift equation (2.24), is there a closed form solution? Or, is there an approximate dynamic system that would converge to the steady state? Otherwise we need to utilize a general numerical solver and write functions for the residuals of each equation, and describe the search space. Note that this is easy enough in a spreadsheet, if coding is not desired.

Closed form seems unlikely. Can we at least approach a linear system with first order terms? Let’s use high and low internal temperatures for variables. Start with the steady heat transfer equations.

If we apply the lift equation to eliminate and use the last equation to substitute , we have two equations in two unknowns. However, COP remains a function of temperature, so the coefficients are non-constant. To higher order, we can include the derivatives of COP since in this ideal, reversible case they are known:

where the superscript star indicates the current state at which the following are also evaluated:

So then we’d have a linear system like this:

This should be able to converge quickly so that we can spend the computing time on optimizing heat exchanger area allocation. Indeed, solving takes about six iterations. Running the optimization is also fast, about 11 ms all together with 44 function evaluations to reproduce example 2.2. (An alternate approach that we haven’t tried here would be to perform entropy generation minimization.)

Anyway, here are the results of the exercise.

|  |
| --- |
|  |
|  |

## 2.7. Lift equation for double-effect cycle

This appears to be impossible to duplicate, since the authors provide neither derivation nor direct reference. In the 1990 paper listed in the references, Herold and Radermacher write:

This relationship is an equality between the temperature lift, from the low to the intermediate temperatures, and the driving temperature difference between the high and the intermediate temperatures. In an actual absorption machine, this relationship is governed by the working fluid characteristics and holds approximately true over wide ranges of operating conditions for both LiBr/H2O and H2O/NH3­.

Nevertheless, the authors themselves provide an answer in example 7.2 (page 147):

## 2.8. Zero-order model COP optimization for realistic U values.

The problem is clearly defined if we continue to treat the absorber and condenser as the same internal temperature. However, the question specifies different overall heat transfer coefficients for absorber and condenser. So we have a choice. One option is to ignore the distinction and add together the absorber and condenser UA values to follow the simple, three resistor model. In this case, obviously we would allocate area to the higher U value component, and the result would be absurd. The other option is to distinguish between heat rejected in the absorber () and condenser (). If we take figure 2.3 as a valid model for single effect cycles, then we can simply take an approximation and (confer tables 6.3 and 6.6). In this case, we still have to decide whether to retain the assumption of equal internal temperatures between those components. However, realistic heat exchange with cooling water would allow different internal temperatures, so let’s try that and write the COP as a function of four variables:

The concept for the single effect lift equation will still be sufficient closure to solve the problem. Meanwhile, looking at the form of the COP gives some insight into a “better” way (for purpose of converging quickly) to parametrize than the temperatures. Looking ahead at the next problem, we might consider using the temperature ratio and the driving temperature (or lift) in each half of the system. Start with the heat engine.

To first order the temperature ratio is

where the reference temperature is the temperature to which both and converge at maximum heat transfer (zero efficiency):

Then the unsigned heat flows are

with the ratio . We thus have a linear system:

To which the solution is

We don’t appear to be making progress with this analysis, so moving on …

## 2.9. Factor with strong effect on COP.

This is a rather poorly defined concept, but the suggested approach is clear enough. Another approach might be to non-dimensionalize the problem and search for principal factors. Or, look at the previous problem and note the form of the efficiencies. Another way to write it is:

Note that if , then .

## 2.10. Maximizing COP.

The result ties back to exercise 2.1.

## 2.11. Double effect cycle: minimum heat input temperature.

This is a direct application of the lift equation from exercise 2.7.